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RTCA Special Committee 186, Working Group 3

ADS-B 1090 MOPS

Meeting #2

Supporting Action Item 1-2:
Analysis of Coast Time Extension

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SUMMARY
The CPR encoding algorithm is much more tolerant to coasting than is currently reflected in the MOPS. This paper seeks to determine and describe exactly what that tolerance is.

Introduction:

This paper seeks to determine the maximum time allowed by the CPR algorithm between the time of applicability of the last known position and the receipt of a new position message before ambiguities may possibly be introduced. For the purposes of initial analysis, we will look at the airborne case. The results may be extrapolated for surface messages, and will be addressed in the conclusion.

After a global decode is achieved, and the receiver has a reference position for his target, the CPR decoding algorithm works in two steps:

1. Determine the latitude

2. Use the latitude to determine the longitude

First, let's look at the latitude. The latitude is encoded using the equation:

$$Y = round\{K \cdot Mod(lat, dlat)\}$$

where K is a precision constant, and Y is the encoding which is transmitted. The important part is that the encoded value comes from Mod(lat, dlat), which is simply the remainder from dividing the actual latitude by dlat. We can let dlat equal 6 for the purposes of this paper. Therefore latitudes which are exactly 6 degrees apart will give the same encoding, e.g.:

latitude	Mod(lat, dlat)	Y
-84.5	5.5	round(5.5*K)
-78.5	5.5	round(5.5*K)
-72.5	5.5	round(5.5*K)
...	...	...
-0.5	5.5	round(5.5*K)
5.5	5.5	round(5.5*K)
...	...	...
83.5	5.5	round(5.5*K)
89.5	5.5	round(5.5*K)

In other words, ambiguous latitudes are 6 degrees apart. There are 30 of these latitudes in 180 degrees (6 * 30 = 180). When the algorithm decodes a received latitude, it chooses one of the 30 possible latitudes based on which one is closest to the last known position on the target (via either a global or local decode). Six degrees of separation between the possible latitudes at ~60 NM/degree corresponds to ~360 NM Therefore, if the transmitting aircraft has gone less than 180 NM in latitude since its position was last successfully decoded, the receiving aircraft will choose the correct latitude.

If the receiver decodes the correct latitude, it now has a chance to decode the correct longitude. The longitude is encoded using much the same equations:

$$X = round\{K \cdot Mod(lon, dlon)\}$$

except that dlon comes from the equation:

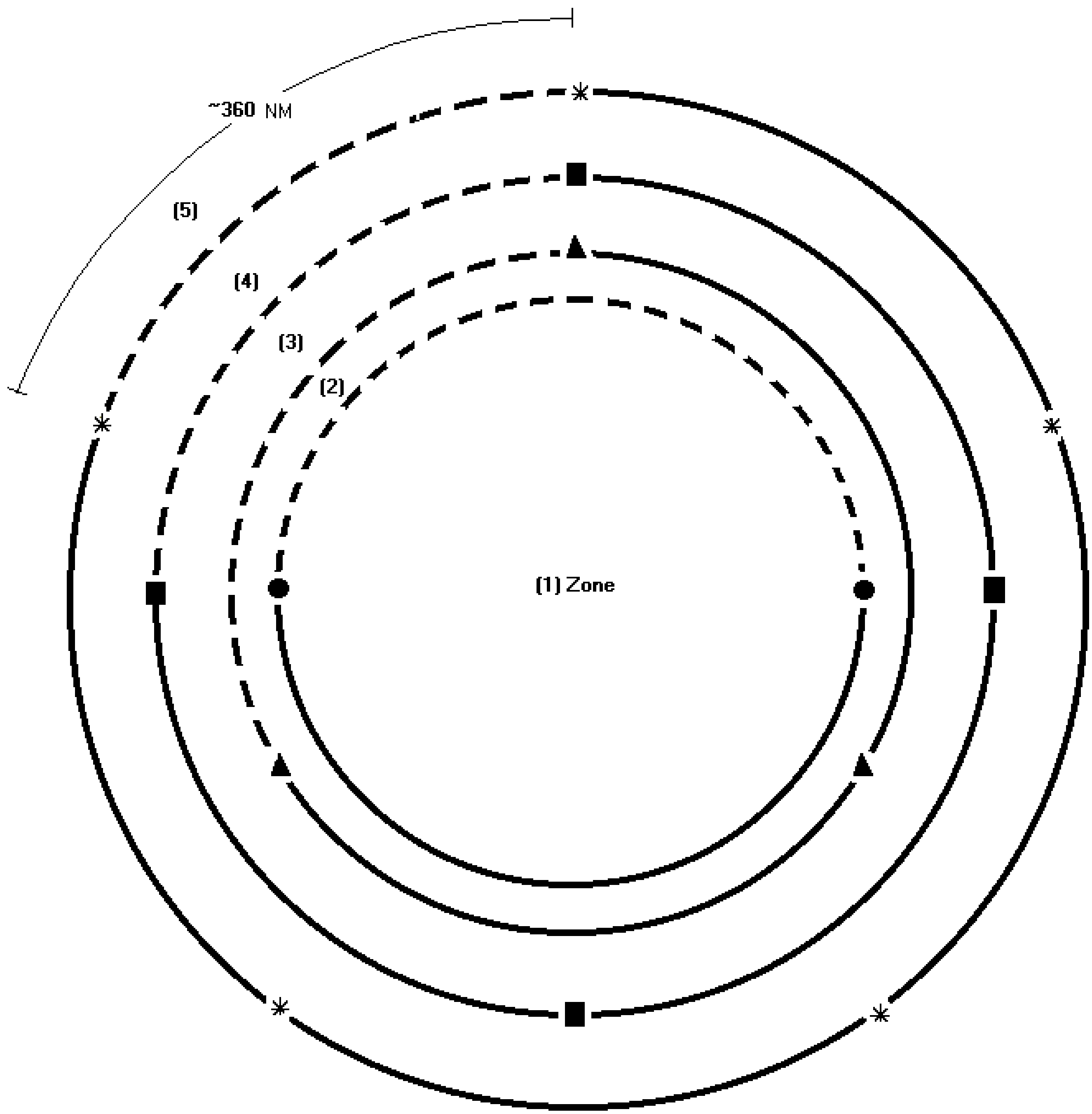
$$dlon = \frac{360}{NL(lat)}$$

The encoded longitude is obtained in the same way as the latitude, except that the number of degrees between ambiguous positions, dlon, varies with latitude. At the equator, NL(0) = 59, making dlon ~6 degrees. One degree of longitude at the equator is ~60 NM, making the separation at the equator 360 NM. Around New Jersey, NL(40) = 45, making dlon ~8. One degree of longitude around New Jersey is $\sim 60 \cdot \cos(40) = 46$ NM. This makes the separation around New Jersey ~360 NM. In fact, the NL function is *defined* to return the maximum number of times you can divide the 360 degrees of longitude and still be guaranteed 360 NM between divisions. Attached is a table providing the minimum separation in NM between ambiguous longitudes for each latitude zone. There is also a representation of the behavior of longitude ambiguities at a pole.

Conclusions:

The CPR algorithm is inherently incapable of reporting changes in position by more than a fixed distance. Specifically what that distance is depends on the parameters set. When using CPR to communicate an airborne position, a target which moves from its last known position by less than 180 NM in the N-S direction, and less than 180 NM in the E-W direction, will be unambiguously located by a receiver with knowledge of that last position. Therefore, the maximum allowable distance for a target to travel unheard from before its position becomes ambiguous is 180 NM. If an aircraft were moving at 1000 knots, 180 NM corresponds to 10.8 minutes. Therefore, the answer to the question, "Can the receiver can wait 120 seconds before it has to do another global decode?", is yes. In fact, there is much more tolerance available.

In the case of surface position decoding, the mathematics are identical to airborne with the exception that there are 4 times as many divisions of the latitude and longitude. This means that there are 4 times as many ambiguous solutions for the decoder to choose from, and the distance between these solutions is 4 times as small (because dlat and dlon are 4 times as small). The distance between latitude solutions is 90 NM, making the maximum change in distance that is visible to the algorithm to be 45 NM. Longitude, likewise, is at a maximum distinguishable distance of 45 NM. At 175 knots (the maximum speed represented in the Surface Position Message), this corresponds to 15 minutes.



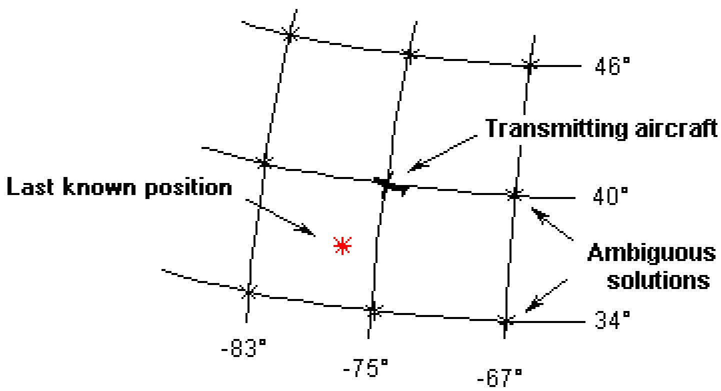
The circles represent the top boundary of a latitude zone. The first circle is the top boundary of zone 2, the second circle of zone 3, etc. The first five zones are labeled. The symbols represent the ambiguous solutions to a single longitude. The shortest path between two ambiguous solutions is along the indicated arcs of longitude. These arcs are never less than 360 NM long.

latitude	NM between solutions		latitude	NM between solutions
0	0		59.9546	366.4977
10.47047	364.5745		61.04918	366.7492
14.82818	363.3748		62.13217	367.0205
18.18626	363.3286		63.20428	367.3138
21.0294	363.3473		64.26617	367.632
23.54505	363.3883		65.31846	367.9781
25.82926	363.4407		66.36171	368.3563
27.93899	363.5006		67.39648	368.7709
29.91137	363.5662		68.42323	369.2274
31.77211	363.6367		69.44243	369.7325
33.53994	363.7117		70.45452	370.2943
35.229	363.791		71.45987	370.9225
36.85026	363.8746		72.45885	371.6298
38.41243	363.9626		73.45177	372.4316
39.92258	364.0553		74.43894	373.3479
41.38653	364.1526		75.42057	374.4044
42.80914	364.2551		76.39685	375.6356
44.19455	364.3629		77.3679	377.0876
45.54627	364.4765		78.33374	378.824
46.86734	364.5961		79.29429	380.9343
48.1604	364.7223		80.24923	383.5507
49.42776	364.8557		81.19802	386.8725
50.67151	364.9966		82.13958	391.2166
51.89343	365.1459		83.072	397.1172
53.09517	365.3042		83.99174	405.5421
54.27818	365.4723		84.89167	418.4288
55.44379	365.6513		85.75542	440.2564
56.5932	365.842		86.53538	484.0179
57.72748	366.0457		87.00001	608.9518
58.84765	366.2638		90	N/A

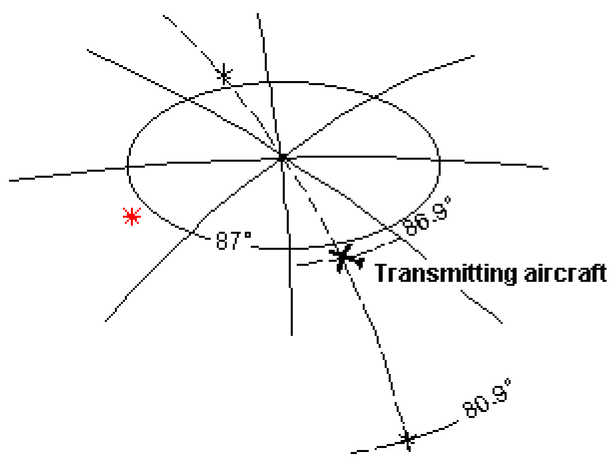
For each line of longitude corresponding to a latitude transition point, the minimum distance between ambiguous solutions along that line (worst case) is given.

Illustrations, to scale, of the nearest ambiguous solutions, upon reception of a single Extended Squitter, even-format

Example 1. Mid-latitude (New Jersey)



Example 2. Near the North Pole



Example 3. Nearer to the North Pole